# Numerical Methods- Questions

June 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

**(2)**

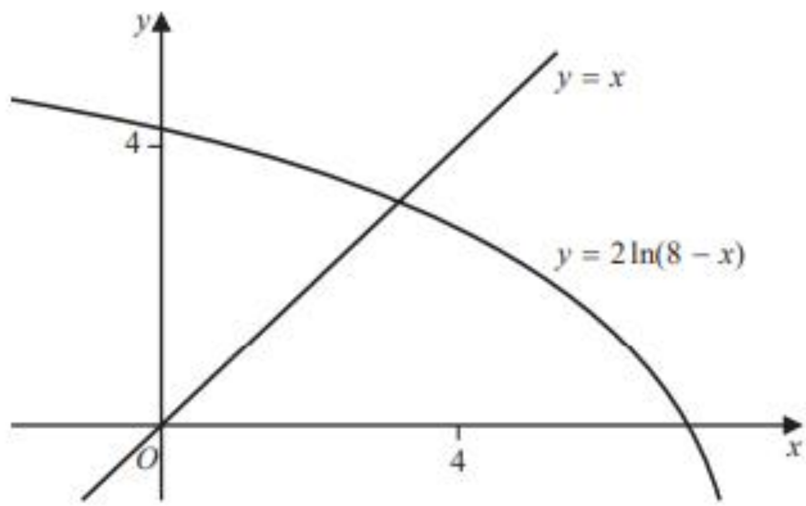


**Figure 2**

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for $\alpha$.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for $\alpha$, justifying your answer.

**(2)**

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2.

**5.**

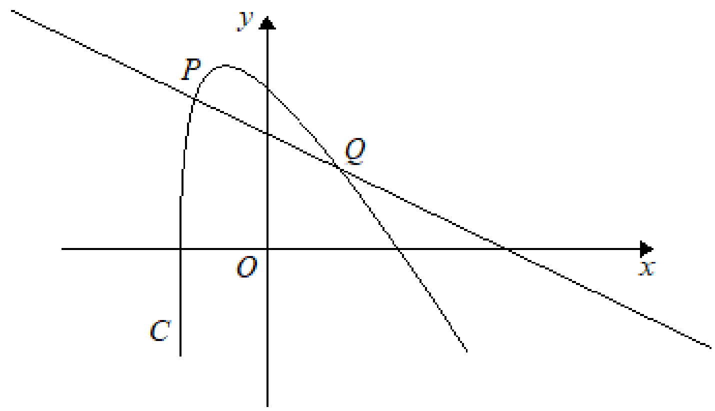


**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point $P$ with $x$ coordinate $-2$ lies on $C$.

($a$) Find an equation of the normal to $C$ at $P$. Write your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers.

**(5)**

The normal to $C$ at $P$ cuts the curve again at the point $Q$, as shown in Figure 2.

($b$) Show that the $x$ coordinate of $Q$ is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

**(3)**

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the $x$ coordinate of $Q$.

($c$) Taking $x_1 = 2$, find the values of $x_2$ and $x_3$, giving each answer to 4 decimal places.

**(2)**

3.

**4.**

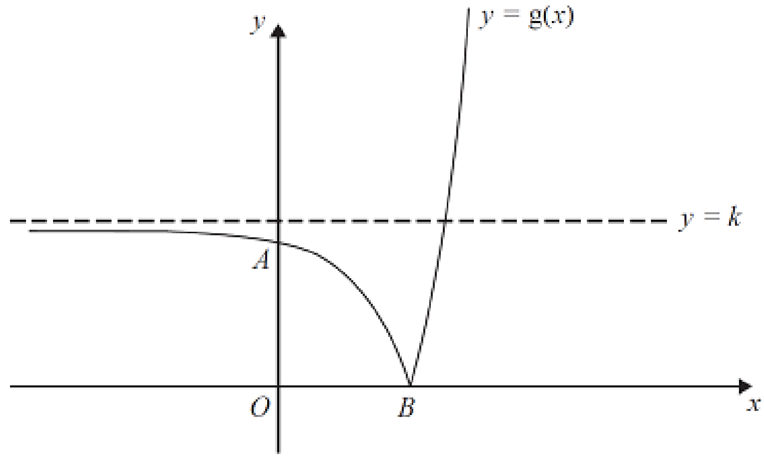


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4\mathrm{e}^{2x} - 25|, \qquad x \in \mathbb{R}.$$

The curve cuts the $y$-axis at the point $A$ and meets the $x$-axis at the point $B$. The curve has an asymptote $y = k$, where $k$ is a constant, as shown in Figure 1.

($a$) Find, giving each answer in its simplest form,

(i) the $y$ coordinate of the point $A$,

(ii) the exact $x$ coordinate of the point $B$,

(iii) the value of the constant $k$.

**(5)**

The equation $g(x) = 2x + 43$ has a positive root at $x = \alpha$.

($b$) Show that $\alpha$ is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$.

**(2)**

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for $\alpha$.

($c$) Taking $x_0 = 1.4$, find the values of $x_1$ and $x_2$. Give each answer to 4 decimal places.

**(2)**

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

**(2)**

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4.

**6.**

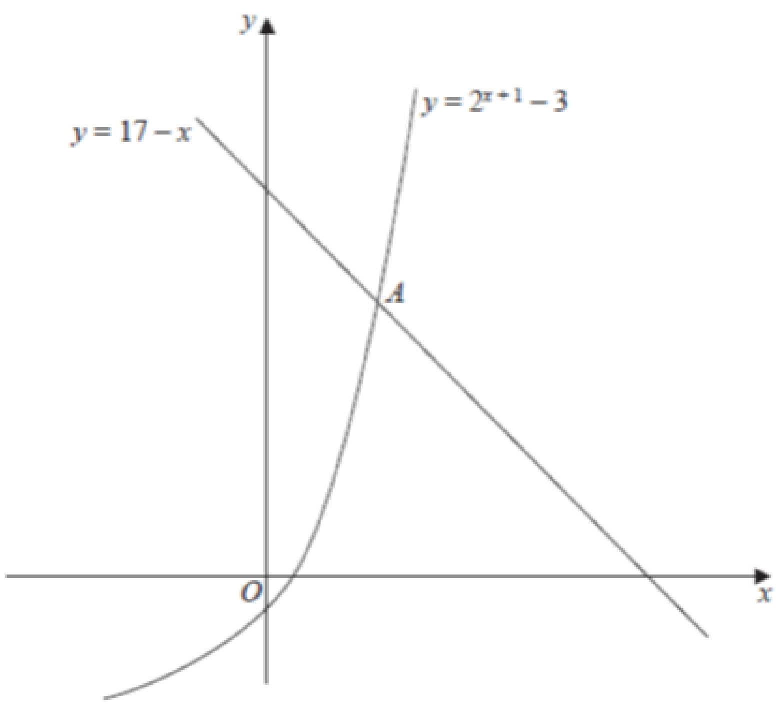


**Figure 1**

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point $A$.

(*a*) Show that the $x$-coordinate of $A$ satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1.$$

**(3)**

(*b*) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \qquad x_0 = 3,$$

to calculate the values of $x_1$, $x_2$ and $x_3$, giving your answers to 3 decimal places.

**(3)**

(*c*) Use your answer to part (*b*) to deduce the coordinates of the point $A$, giving your answers to one decimal place.

**(2)**

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5.

**6.**

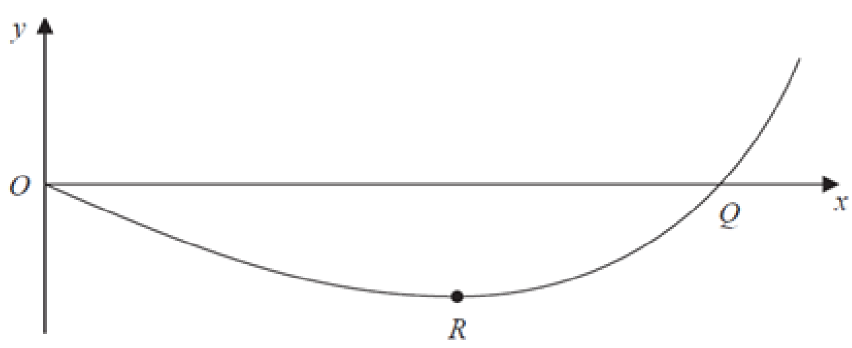


**Figure 2**

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the $x$-axis at the point $Q$ and has a minimum turning point at $R$.

($a$) Show that the $x$ coordinate of $Q$ lies between 2.1 and 2.2.

**(2)**

($b$) Show that the $x$ coordinate of $R$ is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

**(4)**

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n \sin\left(\frac{1}{2}x_n^{\ 2}\right)}, \qquad x_0 = 1.3$$

($c$) find the values of $x_1$ and $x_2$ to 3 decimal places.

**(2)**

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6.

**4.** $$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}.$$

($a$) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

**(5)**

($b$) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$.

**(1)**

The equation $f(x) = 0$ has a root $\alpha$, where $\alpha = 0.5$ to 1 decimal place.

($c$) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of $x_1$, $x_2$ and $x_3$, giving your answers to 3 decimal places.

**(3)**

($d$) Give an accurate estimate for $\alpha$ to 2 decimal places, and justify your answer.

**(2)**

7.

**2.** $$g(x) = e^{x-1} + x - 6$$

(*a*) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6.$$

**(2)**

The root of $g(x) = 0$ is $\alpha$.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2.$$

is used to find an approximate value for $\alpha$.

(*b*) Calculate the values of $x_1$, $x_2$ and $x_3$ to 4 decimal places.

**(3)**

(*c*) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

**(3)**

8.

**2.** $$f(x) = x^3 + 3x^2 + 4x - 12$$

(*a*) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3.$$

**(3)**

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \quad n \geq 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of $x_1$, $x_2$ and $x_3$.

**(3)**

The root of $f(x) = 0$ is $\alpha$.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

**(3)**

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9.

**6.** $$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \qquad 0 \le x \le \pi.$$

(*a*) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$.

**(2)**

The curve with equation $y = f(x)$ has a minimum point $P$.

(*b*) Show that the $x$-coordinate of $P$ is the solution of the equation

$$x = \frac{3 + \sin\left(\tfrac{1}{2}x\right)}{2}.$$

**(4)**

(*c*) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\tfrac{1}{2}x_n\right)}{2}, \qquad x_0 = 2,$$

find the values of $x_1$, $x_2$ and $x_3$ , giving your answers to 3 decimal places.

**(3)**

(*d*) By choosing a suitable interval, show that the $x$-coordinate of $P$ is 1.9078 correct to 4 decimal places.

**(3)**

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10.

**2.** $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \le x < 2\pi.$$

(*a*) Show that $f(x) = 0$ has a root $\alpha$ between $x = 0.75$ and $x = 0.85$.

**(2)**

The equation $f(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}$.

(*b*) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}, \qquad x_0 = 0.8,$$

to find the values of $x_1$, $x_2$ and $x_3$, giving your answers to 5 decimal places.

**(3)**

(*c*) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

**(3)**

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11.

**5.**

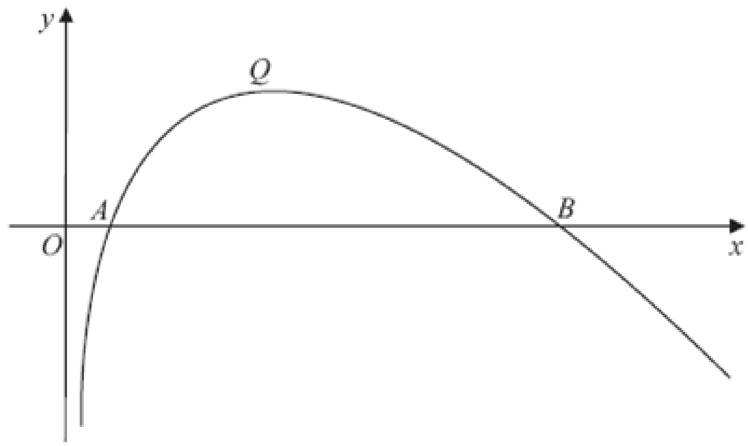


**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x)\ln x, \qquad x > 0.$$

The curve cuts the $x$-axis at the points $A$ and $B$ and has a maximum turning point at $Q$, as shown in Figure 1.

(*a*) Write down the coordinates of $A$ and the coordinates of $B$.

**(2)**

(*b*) Find $f'(x)$.

**(3)**

(*c*) Show that the $x$-coordinate of $Q$ lies between 3.5 and 3.6

**(2)**

(*d*) Show that the $x$-coordinate of $Q$ is the solution of

$$x = \frac{8}{1 + \ln x}.$$

**(3)**

To find an approximation for the $x$-coordinate of $Q$, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(*e*) Taking $x_0 = 3.55$, find the values of $x_1$, $x_2$ and $x_3$.
Give your answers to 3 decimal places.

**(3)**

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12.

**3.** $$f(x) = 4 \operatorname{cosec} x - 4x + 1, \quad \text{where } x \text{ is in radians.}$$

(*a*) Show that there is a root $\alpha$ of $f(x) = 0$ in the interval [1.2, 1.3].

**(2)**

(*b*) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

**(2)**

(*c*) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \quad x_0 = 1.25,$$

to calculate the values of $x_1$, $x_2$ and $x_3$, giving your answers to 4 decimal places.

**(3)**

(*d*) By considering the change of sign of $f(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

**(2)**

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13.

**2.** $$f(x) = x^3 + 2x^2 - 3x - 11$$

(*a*) Show that $f(x) = 0$ can be rearranged as

**(2)**

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

The equation $f(x) = 0$ has one positive root $\alpha$.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$ is used to find an approximation to $\alpha$.

(*b*) Taking $x_1 = 0$, find, to 3 decimal places, the values of $x_2$, $x_3$ and $x_4$.

**(3)**

(*c*) Show that $\alpha = 2.057$ correct to 3 decimal places.

**(3)**